1. If
$$a_n = \frac{1}{(2n-1)(2n+1)}$$
find $S_n = a_1 + a_2 + \cdots a_n$, the sum of the first terms, in closed form (i.e. not as a big long sum). Hence find S_∞.

2. Show that
$$\frac{1}{\sqrt{x+1} + \sqrt{x}} = \sqrt{x+1} - \sqrt{x}.$$
Hence find S_n when
$$a_n = \frac{1}{\sqrt{n+1} + \sqrt{n}}.$$
Determine if
$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} + \sqrt{n}}$$
converges or diverges, and find its value if it converges.

3. Evaluate
$$\sum_{k=1}^{\infty} \left(\frac{1}{4^k} + \frac{1}{5^k} \right)^2.$$