# PHYS 331 — Problem Set #5

**Reading:** Taylor Sections 12.1–12.8

**Reading Journal for Wednesday:** Reaction to Chapter 12.1 – 12.4 material?

**Problems to be handed in Friday September 20:**

1. Taylor 12.3

2. Taylor 12.6. You have already done much of this problem in class, but there are two additional parts:

   (a) Plot your graph that is analogous Fig. 12.3 in the range from $t = 5$ to $t = 6$, and on the same graph plot the function

   $$f_1 = A\cos(2\pi t - \delta)$$

   with $A$ and $\phi$ chosen to make the agreement between the two graphs as good as you can (by eye). Is the numerical solution to the differential equation sinusoidal?

   (b) Repeat the previous part, but this time use the comparison function

   $$f_1 = B\left[\cos(2\pi t - \delta) + C\cos(6\pi t - \delta)\right]$$

   with $B$ and $C$ chosen to make the agreement between the graphs as good as you can. (You can use your previous value of $\delta$.) Revisit the discussion beginning near the top of p. 465 in Taylor. Do your results support Taylor's conclusions?

3. Consider the DDP with $\omega = 2\pi$, $\omega_0 = 3\omega/2$, $\beta = \omega_0/4$, $\phi(0) = -\pi/2$, and $\dot{\phi}(0) = 0$.

   (a) For the value $\gamma = 1.06$:

      i. Make a graph of $\phi(t)$ for times somewhere in the interval $t = 501 \to 600$.

      ii. Make a state space plot for $t = 501 \to 600$.

      iii. Make a Poincaré section for the same time range.

      iv. Are the bifurcation diagram, state space plot, and Poincaré sections consistent?

   (b) Repeat the above for $\gamma = 1.078$.

(c) Repeat the above for $\gamma = 1.081$.

(d) Repeat the above for $\gamma = 1.085$.

4. Consider a DDP with $\omega_0 = 2\pi$, $\omega_0 = 3\omega/2$, $\beta = \omega_0/8$, $\phi(0) = -\pi/2$, and $\dot{\phi}(0) = 0$. Create the Poincare section of Fig. 12.29 (p. 496) for this system.

5. Consider the DDP with $\omega_0 = 2\pi$, $\omega_0 = 3\omega/2$, $\beta = \omega_0/4$, $\phi(0) = -\pi/2$, and $\dot{\phi}(0) = 0$. Make your own version of the bifurcation diagram of Fig. 12.17 on p. 484. (You can use the zooming feature of interactive plots to explore the self-similarity.)

6. Make a bifurcation diagram for a driven damped pendulum that is the same as the system in the previous problem, except this time let $\beta = \omega_0/3.95$. Comment on similarities and differences evident in the two bifurcation diagrams that you have made. Is anything consistent with *universality*?

7. *Predict* the value of $\gamma_5$ for the driven damped pendulum system that was studied in Problem 5. Note that the values of $\gamma_1$ to $\gamma_4$ are given in the caption to Fig. 12.17 (and on p. 475).

8. In Fig. 12.17 I can clearly see period 1 doubling to period 2, period 2 doubling to period 4, and period 4 doubling to period 8, but beyond that I can't really see a "cascade." I want you to make a "high resolution" bifurcation diagram of a very small region of Fig. 12.17 that clearly shows both the doubling of period 4 to period 8, and the doubling of period 8 to period 16. By zooming in appropriately you should be able to get a value of $\gamma_5$ to compare with your previous prediction. Estimate $\gamma_5$ to 5 decimal places. **NOTE:** To get good values for this, you will need to integrate farther than Taylor; I suggest integrating from $t = 0 \rightarrow 2000$, and using the last 100 time values.