PHYS 331 — Problem Set #15

Reading: Taylor 16.6–16.11

Problems to be handed in Tuesday:

1. Consider the illustrated cylinder made of an isotropic elastic material. The bottom of the cylinder is glued to a surface in the x-y plane, and a torque is applied to the top face of the cylinder so that it is rotated an angle α with respect to the bottom face.



- (a) Determine an expression for the displacement vector $\mathbf{u}(\mathbf{r})$ for the material in the cylinder. (It may help to start thinking about this in terms of cylinder polar coordinates; you can switch back and forth between coordinate systems as necessary in the problem.)
- (b) Determine the strain tensor.
- (c) Determine the stress tensor.
- (d) Determine the net torque that top half of the cylinder exerts on the bottom half (at z = L/2). From your result you should be able to see that for a general angular twist Φ of a cylindrical rod of length l, the torque exerted by the cylinder is linear in the angular displacement:

$$|\tau| = \kappa \Phi,$$

where the *torsion constant* is

$$\kappa = S_M \frac{\pi a^4}{l} \Phi.$$

- (e) Show that the net force of the left half of the cylinder (y < 0) on the right half is, as you should expect, equal to zero.
- (f) Determine the speed of torsion waves (waves of the angular displacement) down the cylinder. Does your answer make sense?
- 2. Taylor 16.27
- 3. Taylor 16.31