PHYS 331 — Exam #1

Due 5:00 p.m. on Wednesday October 9

This is a closed-book, closed-notes. Problems 1–6 are to to be completed in one continuous sitting of 75 minutes or less. You are to work alone, without any external assistance. If you have questions, stop the clock and contact me.

Problem 7 is a computer exercise that can be completed separately, with no time limit. It can be turned in via email. It is attached as a separate file.

I have also attached a copy of the information on the inside covers of your book that you can use.

Name:________________________________

Date:________________________________

Location:________________________________

Time Started:________________________________

Time Finished:________________________________

Bucknell Honor Code

As a student and citizen of the Bucknell University community:

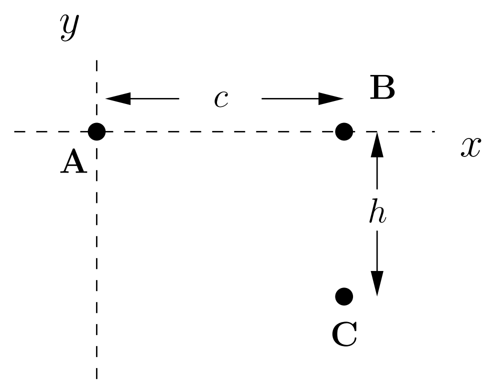
1. I will not lie, cheat, or steal in my academic endeavors.
2. I will forthrightly oppose each and every instance of academic dishonesty.
3. I will let my conscience guide my decision to communicate directly with any person or persons I believe to have been dishonest in academic work.
4. I will let my conscience guide my decision on reporting breaches of academic integrity to the appropriate faculty or deans.

1. (15 pts) Consider a particle with mass m moving horizontally to the right that starts at the origin with initial speed v_0. The only force acting on the particle is a drag force with *magnitude* $f_{\text{drag}} = bv^3$, where v is the magnitude of the velocity. Determine the particle's velocity as a function of time, $v(t)$.

2. (10 pts) Consider a particle with mass m that is thrown straight up from the ground with an initial speed v_0. In addition to the force of gravity, the particle is acted on by drag force with *magnitude* $f_{\text{drag}} = bv^2$, where v is the magnitude of the velocity. Determine a single integral that you can evaluate to determine the height that the mass reaches before changing direction and returning to the ground. You do not need to evaluate the integral, but it should be clear how you would use the results of the integration to determine the height.

3. (15 pts) A vector force in three dimensions given by

$$\mathbf{F}(\mathbf{r}) = r^3\,\hat{\mathbf{r}}.$$



(a) Use cartesian coordinates to demonstrate that this force is conservative.

(b) Do an explicit calculation of the work done by the force along the illustrated path $\mathbf{A} \to \mathbf{C}$. (Use any coordinates you like.)

(c) Do an explicit calculation of the work done by the force along the illustrated path $\mathbf{A} \to \mathbf{B} \to \mathbf{C}$. (Use any coordinates you like.)

4. (15 pts) The logistic map is given by

$$x_{n+1} = rx_n(1 - x_n),$$

where the parameter r is restricted to be between 0 and 4. Tell everything you can about each of the fixed points of this map. Specifically, give:

(a) an expression that gives the value of the fixed point,

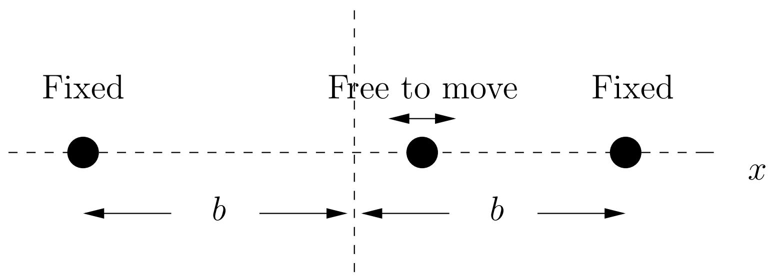
(b) conditions for existence of the fixed point, and

(c) information about the stability of the fixed point.

5. (15 pts) Consider three particles of mass m that interact with a pair-wise repulsive force of *magnitude*

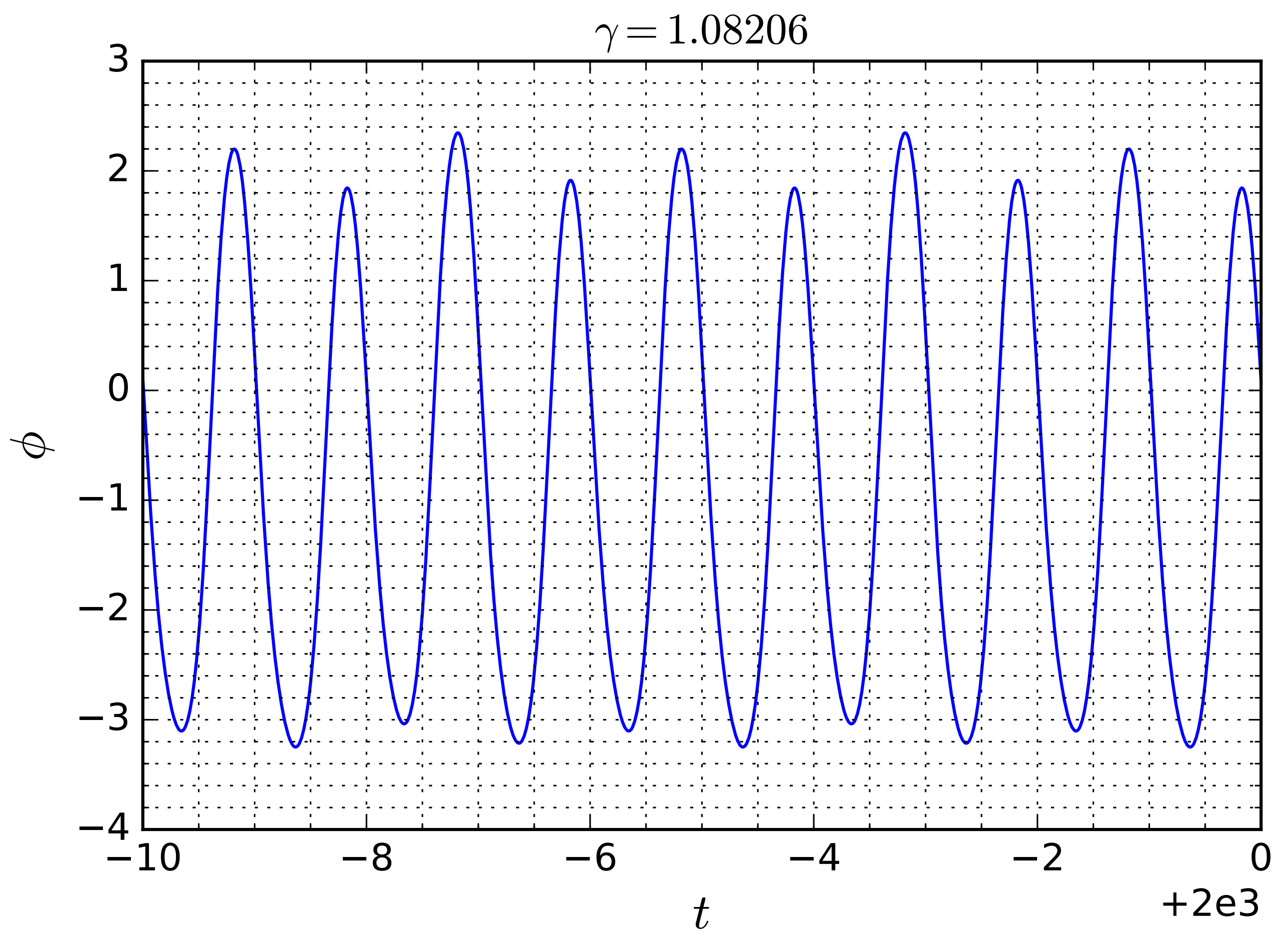
$$F = \frac{c}{r},$$

where c is a constant, and r is the distance between two interacting particles. The particles on the ends are fixed in the indicated positions, while particle in the middle is free to move along the x-axis. Determine the frequency of small oscillations of the middle particle.



6. (15 pts) Below is a graph of ϕ vs. t for a damped oscillator being driven with an angular frequency $\omega = 2\pi$.

Sketch a Poincaré section for this system. While this is only a sketch, it should be possible to read approximate coordinate values for the elements of your graph, and they should be consistent with the graph of ϕ vs. t.



7. (15 pts) For this problem you may use a computer and any programs/notebooks that you have written for this course. Although I don't think it will be necessary, feel free to use the internal python, numpy, and scipy documentation that you can access by putting a '?' after any command. Please do not look for, or use, online examples of how to solve this specific set of equations.

   Consider the following set of three coupled first order differential equations for the time-dependent quantities x, y, and z:

$$\frac{dx}{dt} = \sigma(y - x),$$
$$\frac{dy}{dt} = \rho x - y - xz,$$
$$\frac{dz}{dt} = xy - \beta z,$$

   where ρ, σ and β are constants. For this problem use the constant values $\rho = 28$, $\sigma = 10$, and $\beta = \frac{8}{3}$, and initial conditions $x(0) = y(0) = z(0) = 1$. Find and plot (preferably on the same axes) solutions for $x(t)$, $y(t)$, and $z(t)$ for $t = 0$ to $t = 20$.

Useful Information