3. (20 pts) As a reminder, the equation of motion for a driven damped pendulum is

$$\ddot{\phi} + 2\beta \dot{\phi} + \omega_0^2 \sin \phi = \gamma \omega_0^2 \cos \omega t.$$

The bifurcation diagram for some particular values of ω , ω_0 , and β is shown below.



- (a) Make a qualitative sketch of the kind of ϕ vs. t graph you would get at these times with a drive parameter $\gamma = 1.081$.
- (b) Make a qualitative sketch of the kind of Poincaré section you would get for the same drive parameter.
- (c) Estimate the value of the drive parameter at which the next period doubling will occur as the drive parameter is increased.