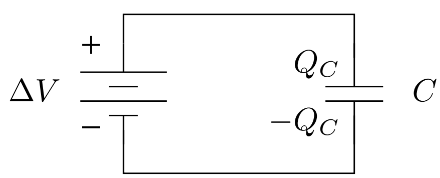
Topic 4

RC Circuits: Response to Step Functions — the *Time Domain*

Material discussed 1/29/19

4.1 Capacitors



- Capacitors are physical breaks in circuits.
- Capactors store an amount of charge that is proportional to the potential difference across the capacitor:

$$Q_C = C\,\Delta V_C \quad \text{or} \quad C = \frac{Q_C}{\Delta V_C}.$$

- SI unit of capacitance is Farads (F).
- Capacitors block DC current; pass (in a sense) AC current.
-

$$\begin{aligned} I &= \frac{dQ_C}{dt} \\ &\to C\frac{dV_C}{dt} \end{aligned}$$

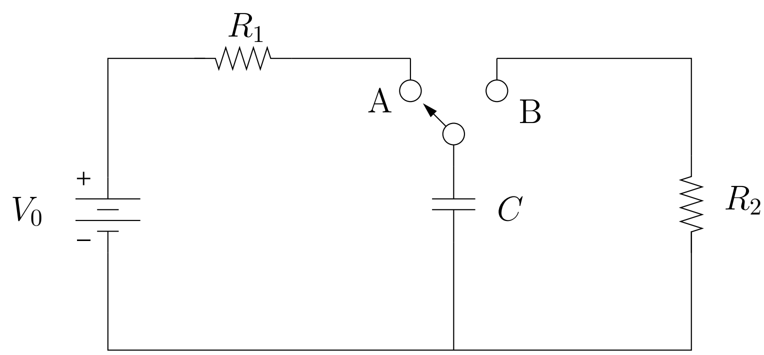


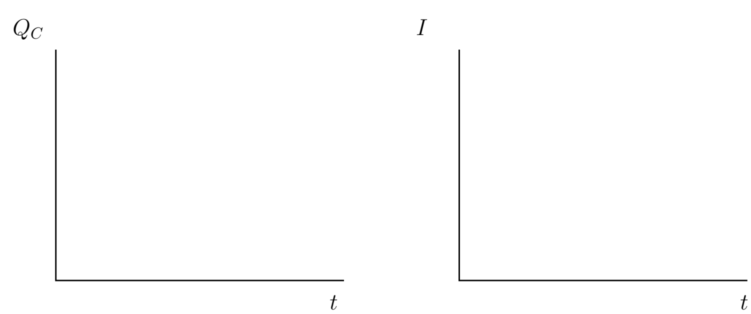
Figure 4.1: Simple RC cicuit.

- Capacitors in parallel:
$$C_{\text{eq}} = C_1 + C_2.$$

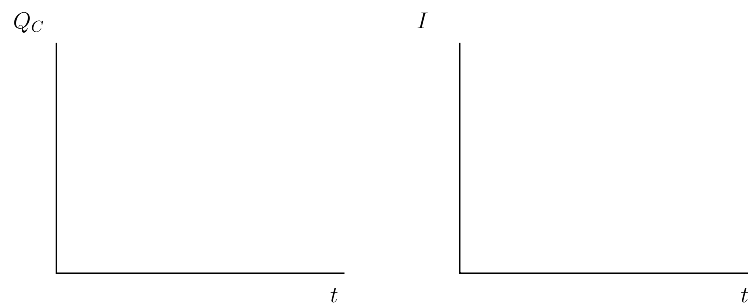
- Capacitors in series:
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2}.$$

4.2 Charging and discharging in RC circuits (qualitative)

- Charging of a capacitor through a resistor:



- Discharging of a capacitor through a resistor:



4.3 Differential Equations

- The solution of a differential equation is a function.
- Functional solutions of differential equations have undetermined constants.
- Undetermined constants can be fixed by considering *initial conditions*.

4.4 Charging and discharging in RC circuits (quantitative)

- Derivations: Kirchoff's voltage law (loop rule) $\longrightarrow$ differential equation.
- Discharging of a capacitor through a resistor
  Kirchoff's voltage law (loop rule):
  $$V_C - IR = 0 \quad \longrightarrow \quad V_C + RC\frac{dV_C}{dt} = 0 \quad \longrightarrow \quad \frac{dV_C}{dt} = -\frac{1}{RC}V_C$$
  Initial Condition:
  $$V_C(0) = V_0 = \frac{Q_0}{C}$$
  Solution:
  $$V_C(t) = V_0 e^{-t/RC} \quad \longrightarrow \quad Q_C(t) = Q_0 e^{-t/RC}.$$
- Charging of a capacitor through a resistor
  Kirchoff's voltage law (loop rule):
  $$V_0 - IR - \frac{Q_C}{C} = 0 \quad \longrightarrow \quad V_0 - RC\frac{dV_C}{dt} - V_C = 0,$$

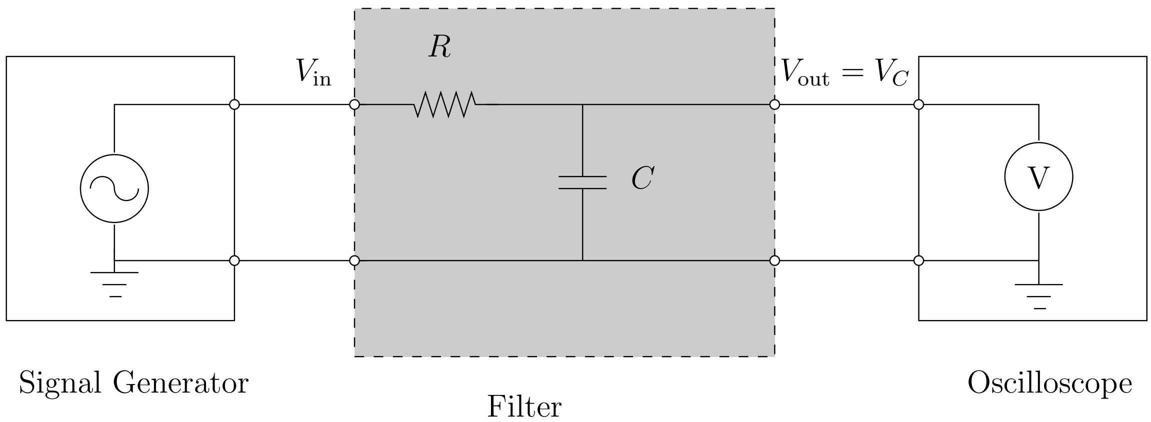


Figure 4.2: Simple RC cicuit with explicitly drawn input and output.

or

$$(V_0 - V_C) + RC\,\frac{d}{dt}(V_0 - V_C) = 0,$$

$$\frac{d}{dt}(V_0 - V_C) = -\frac{1}{RC}(V_0 - V_C).$$

Initial Condition:

$$V_C(0) = 0.$$

Solution:

$$V_C(t) = V_0\left(1 - e^{-t/RC}\right)$$

and

$$V_R(t) = I(t)R = \frac{dQ_C}{dt}R = V_0 e^{-t/RC}.$$

- Characteristic time/decay time/time constant: $\tau = RC$.

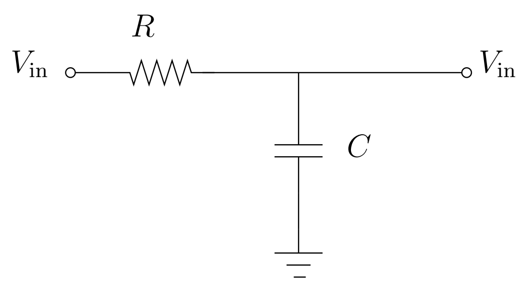


Figure 4.3: Simple *RC* cicuits with implied input and output circuit elements. This circuit is known as an *integrator*.

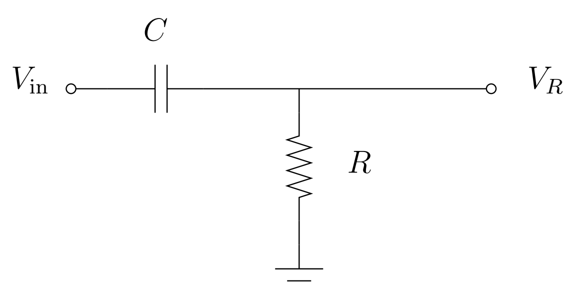


Figure 4.4: Simple *RC* cicuits with implied input and output circuit elements. This circuit is known as a *differentiator*.