Fourier Formulas

Important Integrals

For integer values of m and n:

$$\int_{t_0}^{t_0+T} \sin\left[m\left(\frac{2\pi}{T}\right)t\right] \sin\left[n\left(\frac{2\pi}{T}\right)t\right]\, dt = \begin{cases} T/2 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \tag{1}$$

$$\int_{t_0}^{t_0+T} \cos\left[m\left(\frac{2\pi}{T}\right)t\right] \cos\left[n\left(\frac{2\pi}{T}\right)t\right]\, dt = \begin{cases} T/2 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases} \tag{2}$$

$$\int_{t_0}^{t_0+T} \cos\left[m\left(\frac{2\pi}{T}\right)t\right] \sin\left[n\left(\frac{2\pi}{T}\right)t\right]\, dt = 0 \tag{3}$$

Fourier's Theorem

Any "nice" periodic function $v(t)$ with a period T (and a corresponding fundamental angular frequency $\omega \equiv 2\pi/T$) can by represented by the series

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{m=1}^{\infty} b_m \sin m\omega t. \tag{4}$$

(For a more precise definition of "nice" consult with a mathematician, but suffice it to say that any periodic signal that we come across in physics will be "nice enough.") This representation will also work for any physical signal that extends over a finite interval from some time t_0 to t_0+T. (Outside the interval of interest the Fourier representation will repeat periodically.)

In class we showed that the coefficients are given by:

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} v(t)\cos n\omega t\, dt \tag{5}$$

$$b_m = \frac{2}{T}\int_{t_0}^{t_0+T} v(t)\sin m\omega t\, dt \tag{6}$$