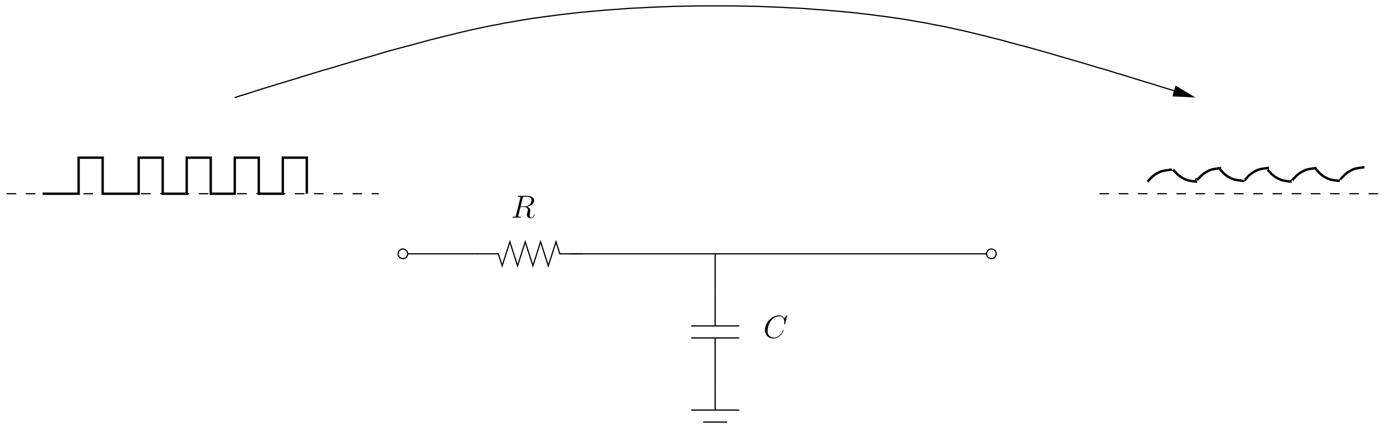
Fourier Series and Filters



$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_1 t + \sum_{m=1}^{\infty} b_m \sin m\omega_1 t$$

$$v'(t) = \frac{a'_0}{2} + \sum_{n}^{\infty} a'_n \cos(n\omega_1 t + \phi_n) + \sum_{m}^{\infty} b'_m \sin(m\omega_1 t + \phi_m)$$

$$a_0 \longrightarrow a'_0$$

$$a_n \longrightarrow a'_n$$

$$b_m \longrightarrow b'_m$$

$$\cos n\omega_1 t \longrightarrow \cos(n\omega_1 t + \phi_n)$$

$$\sin m\omega_1 t \longrightarrow \sin(m\omega_1 t + \phi_m)$$

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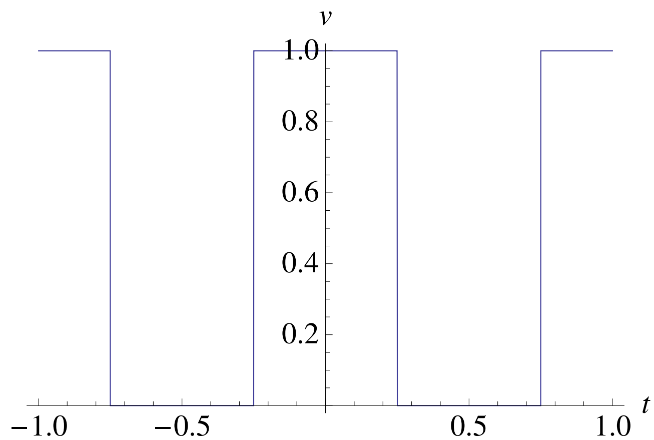
Original Function:



This function repeats periodically outside the illustrated interval. In the interval $-\frac{1}{4} < t \leq \frac{3}{4}$ the function can be written as

$$v(t) = \begin{cases} 1 & \text{for } -\frac{1}{4} < t \leq \frac{1}{4} \\ 0 & \text{for } \frac{1}{4} < t \leq \frac{3}{4} \end{cases}$$

Fourier's Theorem says we can represent the same function as a sum of sines and cosines, with appropriately chosen coefficients:

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega_1 t + \sum_{m=1}^{\infty} b_m \sin m\omega_1 t$$

1. $a_0 =$ __________________________

2. $a_1 =$ __________________________

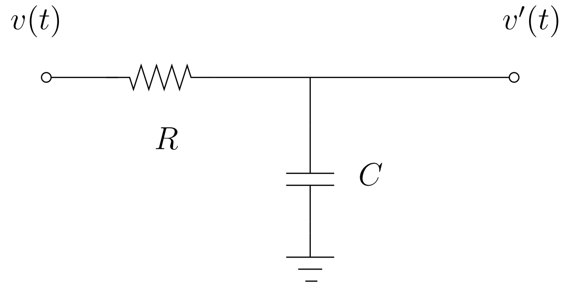
3. $a_2 =$ __________________________

4. $a_3 =$ __________________________

5. $a_n =$ __________________________

6. $b_m =$ __________________________

Filtered Function (Low-Pass):



We know how a low-pass filter affects a sinusoidal input: it reduces the amplitude and it shifts the phase. To see what a low-pass filter does to a square wave, we break it up into its Fourier components, let the filter "act" on each component independently, and then add up the resulting terms. To complete this exercise you will need to remember some things about simple RC circuits.

$$v'(t) = \frac{a'_0}{2} + \sum_{n=1}^{\infty} a'_n \cos(n\omega_1 t + \phi_n) + \sum_{m=1}^{\infty} b'_m \sin(m\omega_1 t + \phi_m)$$

1. $a'_0 =$ ________________________

2. $a'_1 =$ ________________________

3. $\phi'_1 =$ ________________________

4. $a_2' =$ ____________________

5. $\phi_2' =$ ____________________

6. $a_3' =$ ____________________

7. $\phi_3' =$ ____________________

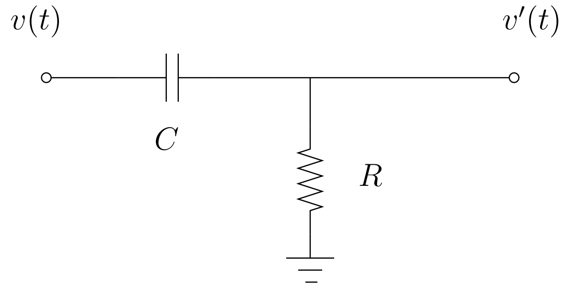
8. $a_n' =$ ____________________

9. $\phi_n' =$ ____________________________

10. $b_m' =$ ____________________________

11. Make a graph of the Fourier sum of the low-pass filtered square wave when $RC = \frac{1}{2}$.

Filtered Function (High-Pass):



$$v'(t) = \frac{a_0'}{2} + \sum_{n=1}^{\infty} a_n' \cos(n\omega_1 t + \phi_n) + \sum_{m=1}^{\infty} b_m' \sin(m\omega_1 t + \phi_m)$$

1. $a_0' =$ __________________________

2. $a_n' =$ __________________________

3. $b_m' =$ __________________________

4. $\phi_n =$ __________________________

5. Make a graph of the Fourier sum of the high-pass filtered square wave when $RC = \frac{1}{10}$.