



Consider linear and ideal transformers attached to Circuit 1 and Circuit 2.

$$\begin{aligned} \mathbf{V}_1 &= (R_1 + j\omega L_1)\mathbf{I}_1 - (j\omega M)\mathbf{I}_2 \\ \mathbf{V}_2 &= (j\omega M)\mathbf{I}_1 - (R_2 + j\omega L_2)\mathbf{I}_2 \\ \mathbf{I}_2 &= \frac{j\omega M}{j\omega L_2 + R_2 + \mathbf{Z}_L}\mathbf{I}_1 \end{aligned} \quad ? \rightarrow ? \quad \begin{aligned} \frac{\mathbf{V}_1}{N_1} &= \frac{\mathbf{V}_2}{N_2} \\ N_1 \mathbf{I}_1 &= N_2 \mathbf{I}_2 \end{aligned}$$

Substitute  $M = k\sqrt{L_1 L_2}$  with  $0 \leq k \leq 1$  and pull out factors in the  $\mathbf{V}_1$  and  $\mathbf{V}_2$  equations:

$$\begin{aligned} \mathbf{V}_1 &= j\omega L_1 \left[ \left(1 + \frac{R_1}{j\omega L_1}\right) \mathbf{I}_1 - k \sqrt{\frac{L_2}{L_1}} \mathbf{I}_2 \right] \\ \mathbf{V}_2 &= j\omega L_1 \sqrt{\frac{L_2}{L_1}} \left[ k \mathbf{I}_1 - \sqrt{\frac{L_2}{L_1}} \left(1 + \frac{R_2}{j\omega L_2}\right) \mathbf{I}_2 \right] \\ \mathbf{I}_2 &= \frac{k}{1 + \frac{R_2 + \mathbf{Z}_L}{j\omega L_2}} \sqrt{\frac{L_1}{L_2}} \mathbf{I}_1 \end{aligned} \quad \rightarrow \quad \begin{aligned} \frac{\mathbf{V}_1}{\mathbf{V}_2} &= \sqrt{\frac{L_1}{L_2}} \frac{\left(1 + \frac{R_1}{j\omega L_1}\right) \mathbf{I}_1 - k \sqrt{\frac{L_2}{L_1}} \mathbf{I}_2}{k \mathbf{I}_1 - \sqrt{\frac{L_2}{L_1}} \left(1 + \frac{R_2}{j\omega L_2}\right) \mathbf{I}_2} \\ \frac{\mathbf{I}_1}{\mathbf{I}_2} &= \sqrt{\frac{L_2}{L_1}} \frac{1 + \frac{R_2 + \mathbf{Z}_L}{j\omega L_2}}{k} \end{aligned}$$

What must occur to achieve the *ideal* transformer relations?

- 1)  $k = 1$ ,  $(\omega L_1) \gg R_1$ ,  $(\omega L_2) \gg R_2$ , and  $\sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2}$  for voltage ratios.
- 2) In addition,  $(\omega L_2) \gg |\mathbf{Z}_L|$  is needed for current ratios.

Comparisons between linear transformer (more accurate) and ideal (when conditions are satisfied):

Open-circuit load:  $\mathbf{Z}_L = \infty$ ,  $\mathbf{I}_2 = 0$

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \sqrt{\frac{L_1}{L_2}} \frac{\left(1 + \frac{R_1}{j\omega L_1}\right)}{k} \quad \text{so} \quad \left| \frac{\mathbf{V}_2}{\mathbf{V}_1} \right| = \sqrt{\frac{L_2}{L_1}} \frac{k}{\left|1 + \frac{R_1}{j\omega L_1}\right|} < \frac{N_2}{N_1}$$

Any finite load:  $|\mathbf{Z}_L| < \infty$

$$\frac{\mathbf{I}_1}{\mathbf{I}_2} = \sqrt{\frac{L_2}{L_1}} \frac{1 + \frac{R_2 + \mathbf{Z}_L}{j\omega L_2}}{k} \quad \text{so} \quad \left| \frac{\mathbf{I}_1}{\mathbf{I}_2} \right| = \sqrt{\frac{L_2}{L_1}} \frac{\left|1 + \frac{R_2 + \mathbf{Z}_L}{j\omega L_2}\right|}{k} > \frac{N_2}{N_1}$$