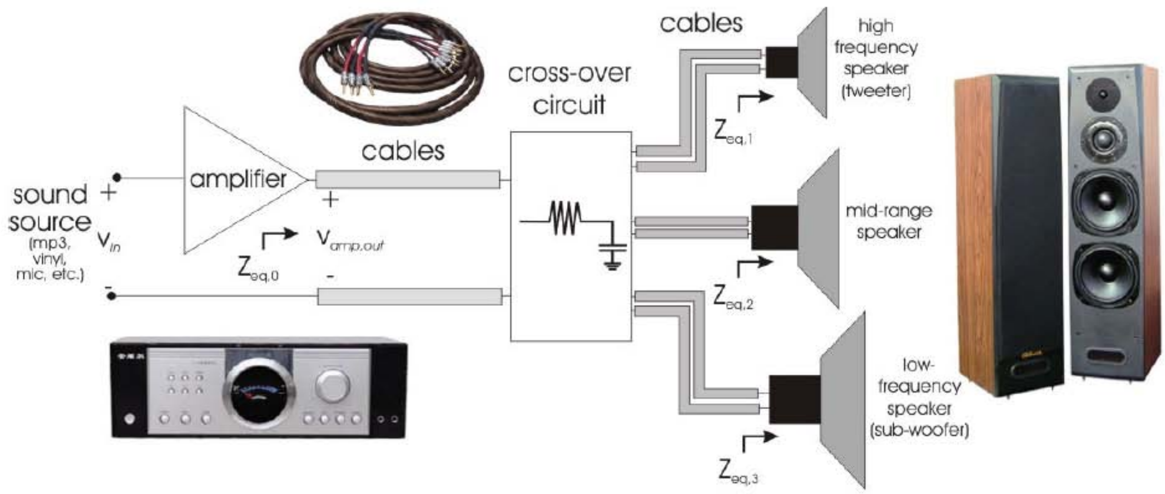
Crossover Network Filter Design

Please read pp. 435-437 on audio applications of filters, and also browse through Chapter 9 for a more detailed discussion. However, all of the required formulas were provided in class and are summarized in these notes.



Our objective is to first design low-pass (LP) and high-pass (HP) filters for an audio system with two speakers, a woofer and tweeter. Then we will design filters for three speakers, so a band-pass (BP) filter is included for a mid-range speaker. The goal is to obtain a "total frequency response" (the superposition of all filters) that is as flat as possible over the range of audio frequencies (so the amplitude gain is close to 0 dB at all frequencies). The total response corresponds to what a listener would hear, because the sounds from all three speakers are added in space.

Each speaker will be modeled as a resistance R = 10 Ω. The LP, HP, and BP filters consist of an inductor (L) and capacitor (C) connected to the speaker (R) as shown in the following circuits.

I do not expect you to carry out the detailed derivations of the equations in these notes. However, the process is identical to what we did in Labs 1 and 2 and Homework 2, but the algebra is more tedious and requires some experience to put the equations in a form that is easier to work with.

Please try to see the value in partitioning the design into two stages:

- *system* design (where we choose general filter parameters such as cutoff frequency and bandwidth), and
- *circuit* design (where we choose circuit component values such as inductance and capacitance for a particular filter circuit).

This partitioning is very useful and common in electrical engineering research and design. Think about how much more difficult the design would be if you had to work with frequency response functions that are complicated functions of ω, L, C, and R!

The LP and HP filters are as follows. (Which is LP and which is HP?)

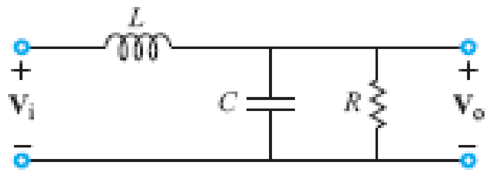


Figure P9.27: Circuit for Problem 9.27.

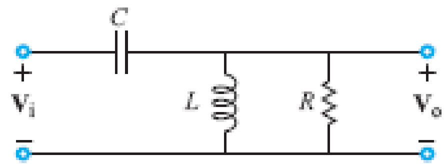


Figure P9.28: Circuit for Problem 9.28.

The voltage divider equation may be rearranged to express the LP and HP frequency response formulas as follows, in terms of the circuit values R, L, and C:

$$H_{\mathrm{LP}}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1 + j\omega \frac{L}{R}(1 + j\omega RC)} \qquad H_{\mathrm{HP}}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1}{1 + \frac{1}{j\omega RC}\left(1 + \frac{1}{j\omega \frac{L}{R}}\right)}$$

The condition for a "maximally flat" magnitude response, $M(\omega)$, is $R = \sqrt{\frac{L}{2C}}$.

This corresponds to damping factor $\xi = 1/\sqrt{2}$ in Figure 9-10 of the textbook.

With this constraint, the cutoff frequency for the LP and HP filters is $\omega_c = \frac{1}{\sqrt{LC}}$, and then the frequency responses can be expressed in terms of the system parameter ω_c as

$$H_{\mathrm{LP}}(\omega) = \frac{1}{\left(1 - \left(\frac{\omega}{\omega_c}\right)^2\right) + j\sqrt{2}\,\frac{\omega}{\omega_c}} \qquad H_{\mathrm{HP}}(\omega) = \frac{1}{\left(1 - \left(\frac{\omega_c}{\omega}\right)^2\right) - j\sqrt{2}\,\frac{\omega_c}{\omega}}$$

Note that at the cutoff frequency, the gain and phase shift for each filter is

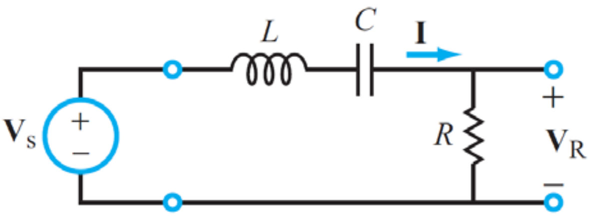
$$H_{\mathrm{LP}}(\omega_c) = \frac{1}{\sqrt{2}}\angle -90^\circ \qquad H_{\mathrm{HP}}(\omega_c) = \frac{1}{\sqrt{2}}\angle +90^\circ .$$

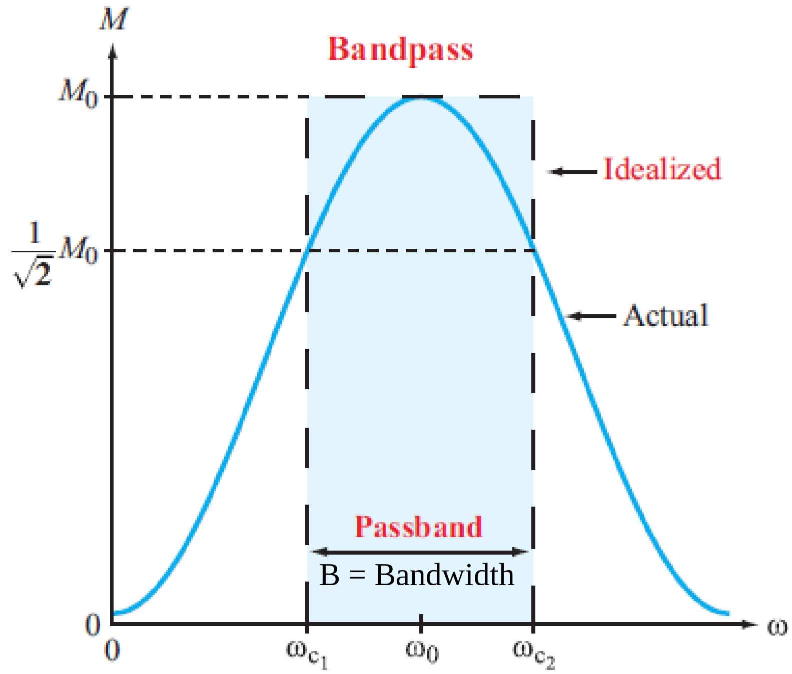
The resistance R is fixed at 10 Ω (for the speaker), but L and C may be chosen to obtain different cutoff frequencies in the LP and HP filters.

The Matlab script `filter_design_LH.m` displays the Bode plot for each filter in Figures 1 and 2. Then Figure 3 displays the frequency response of the total system, $H_{\mathrm{TOTAL}}(\omega) = H_{\mathrm{LP}}(\omega) + H_{\mathrm{HP}}(\omega)$.

This is the sum of complex frequency responses, so the amplitude and phase are both important. The script allows you to adjust the LP and HP filter cutoff frequencies to $\omega_o - (B/2)$ and $\omega_o + (B/2)$, where $\omega_o = 1{,}000$ rad/sec and you can vary $B \in [0, 1000]$.

Band-pass (BP) filter:





The voltage divider equation may be rearranged to express the BP frequency response as

$$H_{\mathrm{BP}}(\omega) = \frac{\mathbf{V}_R}{\mathbf{V}_S} = \frac{1}{1 + j\left(\frac{\omega}{R/L}\right)\left(1 - \frac{1/(LC)}{\omega^2}\right)} = \text{BP freq. resp. in terms of circuit parameters}$$

$$H_{\mathrm{BP}}(\omega) = \frac{1}{1 + j\left(\frac{\omega}{B}\right)\left(1 - \left(\frac{\omega_o}{\omega}\right)^2\right)} = \text{BP freq. resp. in terms of system parameters}$$

The relationships between the BP <u>circuit</u> and <u>system</u> parameters are as follows.

$$\omega_o = \frac{1}{\sqrt{LC}} = \text{frequency where gain is maximum: } H_{\mathrm{BP}}(\omega_o) = 1$$

$$B = \frac{R}{L} = \text{BP filter bandwidth} = \omega_{c,2} - \omega_{c,1}$$

$$\omega_{c,1} = \sqrt{\left(\frac{B}{2}\right)^2 + \omega_o^2} - \frac{B}{2} = \text{lower cutoff frequency}$$

$$\omega_{c,2} = \sqrt{\left(\frac{B}{2}\right)^2 + \omega_o^2} + \frac{B}{2} = \text{upper cutoff frequency}$$

$$\omega_o = \sqrt{\omega_{c,1} \cdot \omega_{c,2}} = \text{geometric mean of lower and upper cutoff freqs.}$$

$$\sqrt{\left(\frac{B}{2}\right)^2 + \omega_o^2} = \frac{\omega_{c,1} + \omega_{c,2}}{2} = \text{arithmetic mean of lower and upper cutoff freqs.} \geq \omega_o$$

$$Q = \frac{\omega_o}{B} = \text{"quality factor" of BP filter (not used in our design)}$$

The Matlab script `filter_design_LHB.m` displays the Bode plot for the LP, BP, and HP filters in Figures 1 and 2. Then Figure 3 displays the frequency response of the total system, $H_{\mathrm{TOTAL}}(\omega) = H_{\mathrm{LP}}(\omega) + H_{\mathrm{HP}}(\omega) + H_{\mathrm{BP}}(\omega)$. The LP cutoff frequency is set equal to $\omega_{c,1}$ of the BP, and the HP cutoff is set equal to $\omega_{c,2}$ of the BP for simplicity.

Filter Design Questions:

The crossover networks will be designed to filter the sound around $\omega_o = 1{,}000$ rad/sec. For two speakers, the woofer will produce sounds at frequencies $\leq \omega_o$ and the tweeter will produce sounds at frequencies $\geq \omega_o$.

For three speakers, the mid-range speaker will produce sounds in a frequency band around ω_o, while the woofer and tweeter will produce the low- and high-frequency sounds, respectively. Your job is to first choose <u>system</u> parameters that produce a total response that is as flat as possible, and then choose <u>circuit</u> parameters for L and C in each filter to realize the desired frequency response.

The constraints are that $B \leq 1{,}000$ rad/sec and all inductor values are between 1 mH and 100 mH. In addition, the LP and HP filters must be "maximally flat," which requires $R = \sqrt{\dfrac{L}{2C}}$ where R is fixed at 10 Ω (for the speaker).

1. Consider the design of a crossover network for two speakers using LP and HP filters in the crossover network. Use the Matlab script `filter_design_LH.m` .
    a. What is the slope of the LP and HP filters in the stop-band of the magnitude response Bode plot?
    b. With B = 0, why is the gain of the total system small at $\omega_o = 1{,}000$ rad/sec?
    c. Try changing B in the range 0 to 1,000 rad/sec and observe the effect on the total response. What value for B gives the flattest gain over the audio frequency range (10 to 20,000 Hz)? Look at the Bode plots for the LP and HP filters as B is varied and try to understand the effect on the total response. (You don't have to choose <u>circuit</u> parameters for this case.)
2. Consider the design of a crossover network for three speakers using LP, BP, and HP filters in the crossover network. Use the Matlab script `filter_design_LHB.m` .
    a. What is the slope of the BP filter in each stop-band of the magnitude response (that is, frequencies $< \omega_{c,1}$ and $> \omega_{c,2}$)?
    b. Try changing B up to a maximum of 1,000 rad/sec and observe the effect on the total response. What value for B gives the *flattest* gain over the audio frequency range (use your judgment, this is subjective)? Look at the individual Bode plots for the LP, HP, and BP filters as B is varied and try to understand the effect on the total response.
    c. For B that gives the flattest response, what are the filter cutoff frequencies?
    d. Design LP, BP, and HP filters to realize your "best" total response. That is, specify <u>circuit</u> values for L and C in each filter to achieve the desired total response. (Each filter requires different L and C values!)
    e. Test your values for L and C (for each filter) in the Matlab script `filter_RLC.m` and verify that the desired frequency response is achieved.